DILATATION

1. $\alpha_L \simeq 1{,}5 \cdot 10^{-5}\,K^{-1}$ $\qquad \left(\dfrac{\Delta\ell}{\ell_0 \cdot \Delta\theta} = \alpha_L\right)$

2. a) $\ell_{40} \simeq 20{,}006\ m$ $\qquad (\ell = \ell_0 (1+\alpha_L \cdot \Delta\theta))$

b) $\theta' \simeq 35°C$ $\qquad \left(\theta' = 20°C + \Delta\theta\ ;\ \Delta\theta = \dfrac{\Delta\ell}{\alpha_L \cdot \ell}\right)$

3. $d_0 \simeq 1{,}68\,m$ $\qquad \left(\ell_0 = \dfrac{\ell}{1+\alpha_L \cdot \Delta\theta}\ ;\ \pi \cdot d_0 = \dfrac{\pi \cdot d}{1+\alpha_L \cdot \Delta\theta}\ \text{soit}\ \boxed{d_0 = \dfrac{d}{1+\alpha_L \cdot \Delta\theta}}\right)$

4. $\Delta\ell \simeq 6 \cdot 10^{-6}\,m$ soit $\Delta\ell \simeq 1{,}5\,\mu m$ $\qquad \left(\Delta\ell = \dfrac{F \cdot \ell}{E \cdot S}\right)$

5. De 20°C à 40°C, la Tour Eiffel se dilate
De 20°C à −10°C, elle se contracte

$$\ell_{-10} = \frac{\ell_{20}}{1+\alpha_L \cdot \Delta\theta_2} \qquad \ell_{40} = \ell_{20} \cdot (1+\alpha_L \cdot \Delta\theta_1)$$

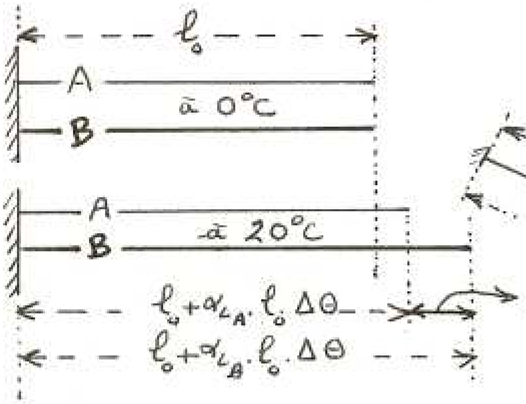
variation de hauteur entre −10°C et +40°C

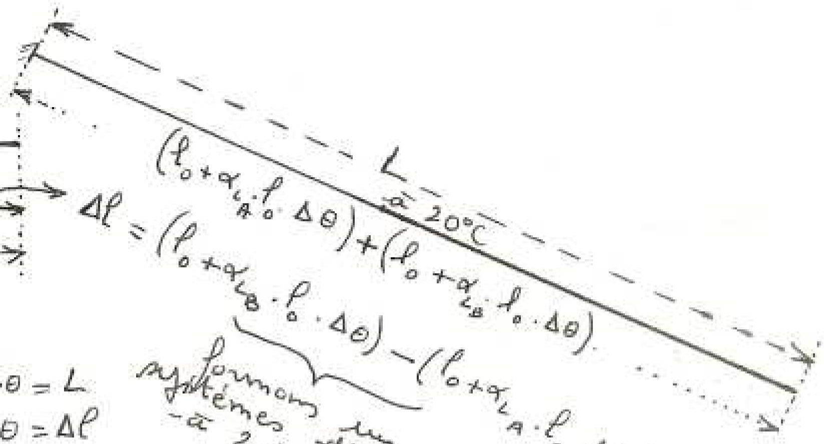
$$\Delta\ell = \ell_{40} - \ell_{-10}$$

$$\boxed{\Delta\ell = \ell_{20}\left[(1+\alpha_L \cdot \Delta\theta_1) - \left(\frac{1}{1+\alpha_L \cdot \Delta\theta_2}\right)\right]}$$

$\Delta\ell \simeq 18\,cm$.

6.





$(\ell_0 + \alpha_{L_A} \cdot \ell_0 \cdot \Delta\theta) + (\ell_0 + \alpha_{L_B} \cdot \ell_0 \cdot \Delta\theta)$

$\Delta\ell = \underbrace{(\ell_0 + \alpha_{L_B} \cdot \ell_0 \cdot \Delta\theta) - (\ell_0 + \alpha_{L_A} \cdot \ell_0 \cdot \Delta\theta)}$

formons un système de 2 équations à 2 inconnues α_{L_A} et α_{L_B}

$$\begin{cases} \ell_0 + \alpha_{L_B} \cdot \ell_0 \cdot \Delta\theta + \ell_0 + \alpha_{L_A} \cdot \ell_0 \cdot \Delta\theta = L \\ \cancel{\ell_0} + \alpha_{L_B} \cdot \ell_0 \cdot \Delta\theta - \cancel{\ell_0} - \alpha_{L_A} \cdot \ell_0 \cdot \Delta\theta = \Delta\ell \end{cases}$$

(après de rudes efforts mathématiques :)

$\alpha_{L_A} \simeq 2{,}3 \cdot 10^{-5}\,K^{-1}$

$\alpha_{L_B} \simeq 2{,}9 \cdot 10^{-5}\,K^{-1}$

7. a)

$\ell = \ell_0 (1+\alpha_L \cdot \Delta\theta)$

$S_0 = \ell_0^2$

$S = \ell^2 = [\ell_0 + \alpha_L \cdot \ell_0 \cdot \Delta\theta]^2$

$= \ell_0^2 + 2\alpha_L \cdot \ell_0^2 \cdot \Delta\theta + \cancel{\alpha_L^2\, \ell_0^2\, \Delta\theta^2}$ $\simeq 0$ à cause surtout de $\alpha_L^2\ (\simeq (10^{-5})^2 = 10^{-10})$

$S \simeq S_0 + 2\alpha_L \cdot \ell_0^2 \cdot \Delta\theta$

$\Delta S = S - S_0 \simeq \underbrace{2 \cdot \alpha_L}_{\alpha_S} \cdot S_0 \cdot \Delta\theta$

b) cube $\quad V_0 = \ell_0^3$

$V = \ell^3$

$\Delta V = V - V_0 \simeq \underbrace{3 \cdot \alpha_L}_{\alpha_V} \cdot V_0 \cdot \Delta\theta$

8. $\Delta S = \alpha_S \cdot S_0 \cdot \Delta\theta$ $(\alpha_S = 2\alpha_L)$ $\qquad \Delta S = 2 \cdot \alpha_L \cdot S_0 \cdot \Delta\theta \qquad \Delta S \simeq 0{,}0028\,m^2$ (28 cm^2)

9. $S = S_0\,(1 + 2\alpha_L \cdot \Delta\theta) \qquad S \simeq 0{,}1965\,m^2$

10. a) $\Delta S \simeq 0{,}048\,cm^2$ $(4{,}8\,mm^2)$ $\qquad \Delta V \simeq 0{,}0234\,cm^3$ $(23{,}4\,mm^3)$

b) $S_{100} \simeq 12{,}61\,cm^2 \qquad V_{100} \simeq 4{,}2127\,cm^3$

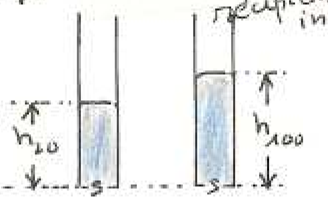
11. a) Surface latérale $\simeq 1{,}4137\,m^2$

Volume $\simeq 0{,}1060\,m^2$

b) $S_{40} \simeq 1{,}4143\,m^2$ et $V_{40} \simeq 0{,}1061\,m^3$

12. contraction $\quad C(-30) = \dfrac{C(20)}{1 + 3 \cdot \alpha_L \cdot \Delta\theta}$ $(\Delta\theta = 70°C)$ $\qquad C(-30) \simeq 998{,}2\,L < 1000\,L$

14.



dilatation volumique

$V_{100} = V_{20} \cdot (1 + \alpha_V \cdot \Delta\theta)$

$\cancel{S}\,h_{100} = \cancel{S}\,h_{20}\,(1 + \alpha_V \cdot \Delta\theta)$

$\boxed{h_{100} = h_{20}\,(1 + \alpha_V \cdot \Delta\theta)}$

$h_{100} \simeq 16{,}23\,cm$

15. $V_{80} = V_0\,(1 + \alpha \cdot \Delta\theta)$

$V_{80} \simeq 29{,}0\,L \qquad (\Delta V \simeq 6{,}6\,L)!$

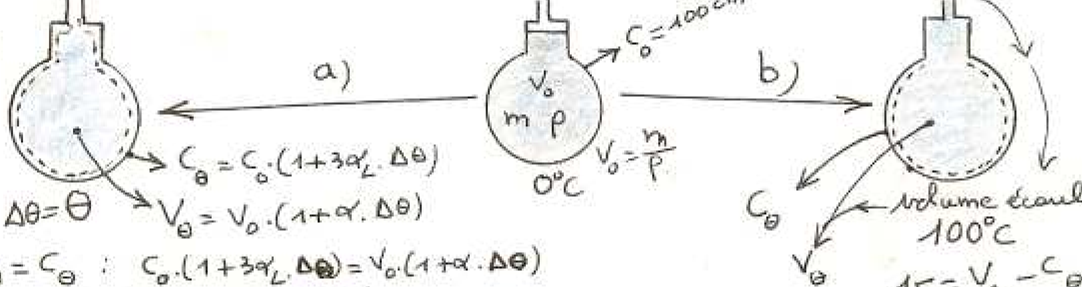
16. $V_{écoulé} = \Delta V_{mercure} - \Delta V_{récipient}$

$\qquad \alpha \cdot V_0 \cdot \Delta\theta \;-\; 3 \cdot \alpha_L \cdot V_0 \cdot \Delta\theta \qquad \left(V_0 = \dfrac{m}{\rho}\right)$

$\boxed{V_{écoulé} = \dfrac{m}{\rho} \cdot \Delta\theta \cdot (\alpha - 3 \cdot \alpha_L)}$

$V_{écoulé} \simeq 0{,}805\,cm^3$

17.



$V_\theta = C_\theta \;:\; C_0 \cdot (1 + 3\alpha_L \cdot \Delta\theta) = V_0 \cdot (1 + \alpha \cdot \Delta\theta)$

$\boxed{\Theta = \dfrac{C_0 - V_0}{V_0 \cdot \alpha - 3 \cdot \alpha_L \cdot C_0}}$

$\Theta \simeq 58{,}5°C$

$v = V_\theta - C_\theta$

$\boxed{v = V_0\,(1 + 100 \cdot \alpha) - C_0\,(1 + 300 \cdot \alpha_L)}$

$v \simeq 0{,}636\,cm^3$

c) $v = V_0 \cdot (1 + \alpha \cdot \Theta) - C_0 \cdot (1 + 3 \cdot \alpha_L \cdot \Theta)$

$v = \Theta\,(\alpha \cdot V_0 - 3 \cdot \alpha_L \cdot C_0) + (V_0 - C_0)$

$\boxed{v = 0{,}01534 \cdot \Theta - 0{,}84798}$

Extraits BTS !...

TP 1991

1. $T = 2\pi \sqrt{\frac{\ell}{g}}$

$T \simeq 3,173495$

2. sous l'effet de la dilatation, le câble s'allonge

$\ell' = \ell \cdot (1 + \alpha_L \cdot \Delta\theta)$

$\ell' = 2,5006 m$

$T' = 2\pi \cdot \sqrt{\frac{\ell'}{g}}$

$T' \simeq 3,173878$

3. incertitude relative $\frac{\Delta T}{T} \sim 1,2‰_0 \; (0,012\%)$

X1

1. $\boxed{\Delta\ell = \alpha_L \cdot \ell \cdot \Delta\theta}$ $\quad \Delta\ell \simeq 1,8 \cdot 10^{-3} m$ — $1,8 mm$

2. $\Delta\ell = \frac{F \cdot \ell_0}{E \cdot S}$ $\quad \boxed{F_{30} = \frac{E \cdot S \cdot \Delta\ell}{\ell_{15}}}$ $\quad F_{30} \simeq 3600 N$ — $3,6 kN$

3. $P = \frac{F}{S}$ $\quad P_{30} = \frac{E \cdot \Delta\ell}{\ell_{15}}$ $\quad P_{30} \simeq 36\,000\,000 Pa$ — $36 MPa$

c'est énorme !!!

4. $P_{30} >> P_{atm} \; (100\,000 Pa)$

5. $\alpha_{L\,béton} \simeq \alpha_{L\,acier}$ $\longrightarrow$ les joints de dilatation permettent "l'agrandissement" des plaques sans risque de fissures ... rires !!!